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PREDICTIVE CONTROL OF PLASMA KINETICS: TIME-RESOLVED MEASUREMENTS OF INERT GAS MIXING IN A HOLLOW CATHODE DISCHARGE

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Improved Hall-effect Thruster (HET) efficiency would enable more extensive cost-capped, Discovery class NASA missions such as robotic missions to Mars and near-Earth asteroids to perform round trip sample-returns. HET efficiency would be further improved if electrons with energies that contribute to ionization were increased, and those involved in transient processes were reduced. Therefore, control of the electron energy distribution functions (EEDFs) is needed. Inert gas mixing is proposed to be an effective method for tuning electron temperature and density. An initial assessment of this method for EEDF control in HETs shows that when 1% of the total gas flow in a hollow cathode's argon plasma is substituted with nitrogen gas, the electron temperature increases from 3.2 eV to 3.8 eV and the number density decreases from $5 \times 10^{17} \text{ m}^{-3}$ to $2.2 \times 10^{17} \text{ m}^{-3}$. The EEDF for the case with nitrogen gas mixing showed more distinct plateaus and peaks compared to the case with a pure argon plasma. This analysis will be used to guide further investigation into using a gas-mixing method to demonstrate predictive electron energy control in HETs. The end goal is to develop schemes to predictably tailor the EEDF in order to increase HET efficiency.

I. INTRODUCTION AND MOTIVATION

With the continued push for more efficient use of propellant in HETs, better control of the electron energy distribution functions (EEDFs) is needed. This is because, in order to make a plasma thruster more efficient, electrons with energies that contribute to the ionization of neutrals need to be increased, and those that are involved in transient processes, or that do not have enough energy to ionize need to be decreased. However, the ability to predictably control the EEDFs of low temperature plasmas (LTPs) remains a challenging problem in plasma physics due to the complex electromagnetic interactions that take place in the actual system.

In a Hall-effect Thruster (HET), the anode (at the beginning of the thruster channel) and the cathode (just outside the exit plane) set up an axial electric field. In addition, a semi-radial magnetic field is applied towards the end of the thruster channel using anti-parallel, coaxial solenoids of different lengths. This $\mathbf{E} \times \mathbf{B}$ field causes electrons from the cathode to circulate azimuthally near the exit plane, establishing the "Hall current." Neutral gas propellant that is released into the channel collides with this high-energy Hall current. The resultant ions are accelerated out of the thruster by the electric field. While the general operation of HETs is fairly well understood, fundamental physical issues that

affect thruster efficiency remain unresolved. The efficiency of HETs depends on their ability to ionize and accelerate propellant to high exhaust velocities. However, the interactions between the ionized propellant particles, electrons, and the applied electromagnetic fields give rise to an electromagnetically dynamic exhaust plasma plume with a broad spectrum of oscillations carried by waves of ionized propellant. The oscillations with the highest amplitude have frequencies less than 1-MHz.*

Previous research gives evidence that these low-frequency oscillations are one of the causes of the anomalously high transport of electrons across the applied magnetic field. This "cross-field" electron transport, which is observed to be 10-20 times larger than what classical collision theory predicts, is the axial drift of the electrons from the cathode, across regions of strong magnetic field towards the anode at the beginning of the thruster channel. These electrons reach the anode without ionizing neutral propellant, which uses up power and therefore reduces thruster efficiency.† The hypothesis is that if this cross-field transport is reduced, the HET's efficiency will increase. The reason why this is not a straightforward assertion is because of the dynamic nature of the system, which causes a number of factors to change if one thing is altered. Therefore, the interaction between these

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electrons and the oscillations needs to be understood, along with how this interaction is affected by the other aspects of the system, in order to determine how to predictably control the EEDF.

The overall goal of this research is to develop methods for tailoring the EEDF to affect these parameters in a way that increases thruster efficiency and lifetime in the low-power operating range. As stated in NASA's *Human Exploration and Development of Space Enterprise*, one of the agency's objectives is to invest "in the development of high-leverage technologies to enable safe, effective and affordable human/robotic exploration." If efficiency and lifetime can be further increased for HET operation in the low-power range, this will enable more complex and extensive cost-capped, Discovery class science missions. Proposed science missions that could be facilitated with higher performance propulsion systems include robotic missions to the Moon, Mars, and near-Earth asteroids to perform round trip sample-returns, to search for and prepare resources, to set up sites in anticipation of future landings, and to demonstrate new technology in preparation for even more challenging and longer-range robotic missions.[‡]

The initial findings from the proposed method of EEDF control, in which two inert gases are used for generating the plasma, will be discussed in this paper.

II. PLASMA SOURCE AND DIAGNOSTIC

A lab-built Lanthanum Hexaboride (LaB_6) cathode was developed at the Plasmadynamics and Electric Propulsion Laboratory (PEPL)[§] for carrying out various preliminary experiments to study electron dynamics in Hall-effect thrusters (HETs). The hollow cathode is the main source of electrons for the thruster, and can be operated with a smaller scale production and a shorter turn-around time. Therefore, this cathode serves as an ideal platform for testing the experimental methodology of various electron dynamics studies for HETs, and for obtaining preliminary results. This lab-built cathode was used as the plasma source in this initial gas-mixing experiment.

Triode Mode

The cathode's dimensions and overall design were chiefly based on high-current LaB_6 cathodes developed for use with the 6-kW laboratory model Hall thruster.^{**,*†} The cathode was run in triode configuration with an external annular anode, mimicking a conventional Hall thruster, but without anode flow (Fig. 1). This configuration draws the plasma outside of the cathode and creates a more stable plume. For both of the gas-mixing cases studied in this initial test, the total flow rate was 25 sccm, the keeper current was limited to 6 A and anode voltage was set to 32 V (Table 1).

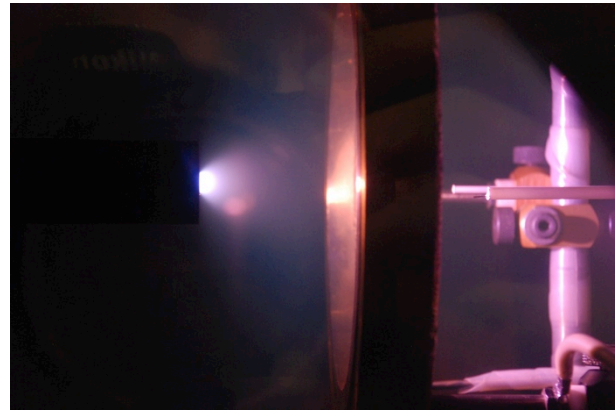


Fig. 1: Operation of Cathode in Triode Mode.

This figure shows the cathode plume in triode mode. The plasma fans out towards the edges of the anode and is noticeably brighter in this mode. The Langmuir probes and the anode's support structure are seen on the far right in this photo.

Time-resolved Langmuir Probe Measurements

A lab-built high-speed dual Langmuir probe (HDLP) system was used to obtain time-resolved measurements of electron density, electron temperature, plasma potential, and the EEDF. This HDLP system was built to obtain clear, high frequency Langmuir probe (LP) traces that can be individually analyzed to obtain time-resolved results. The system uses a custom-built, high bandwidth, high-voltage amplifier. It also has a current sensing circuit that features thin-film metal oxide, ultra-low inductance shunts, and high-bandwidth, ultra-high input impedance voltage and current JFET (junction field-effect transistor) buffers.^{**} For the experiment in this paper, a sweep rate of 15-kHz along with a data acquisition (DAQ) system with a 1-MHz maximum sampling rate were used. However, an upgraded version of the HDLP system has also been developed in the lab with an unprecedented 1- μs resolution. This system will be used in future tests of this and other EEDF control methods.^{§§,***}

III. GAS-MIXING EXPERIMENTAL SETUP

In other low temperature plasmas (LTPs), inert gas mixing has been shown to be an effective method for tuning the electron temperature and density by almost an order of magnitude.^{†††} In addition, simulations have predicted that when one of these gases is a small percentage of the mixture, its naturally present, small fraction of metastable states have a greater effect on the plasma; and the superelastic collisions that ensue create plateaus, holes and peaks in the EEDF.^{†††}

The gas-mixing experiment that will be discussed in this paper was carried out at the Plasmadynamics and Electric Propulsion Laboratory (PEPL) in the Cathode Test Facility (CTF). This cylindrical vacuum chamber is

2-m long and 0.6-m in diameter (Fig. 2). The chamber's base pressure was 1×10^{-7} Torr for these tests.

Table 1 shows the cathode's operating conditions for the two cases examined in the analysis. In order for the cathode to maintain steady operation, the current the anode drew and the potential difference that was established between the keeper and cathode were slightly different in the two cases. These differences can be attributed to the different gas compositions, which affects the plasma parameters as well. This will be seen in the Langmuir probe analysis. Unit Flow Controllers (UFCs) from the 7300 series were used to establish and maintain the argon and nitrogen gas flows. The error bars for the gas flow shown in the table were calculated using the specification sheets for these controllers.

The cathode's heater was also used to help maintain the operating condition during the tests. A higher heater current was needed to maintain the plasma discharge in the case with nitrogen gas mixing. This is not due to the common misconception that molecular gases would have a higher ionization energy. In fact, the ionization threshold energy of N_2 is around 15.6 eV, which is comparable to that of argon at 15.78 eV. In addition the dissociation energy of molecular nitrogen is only a few eV in a low-pressure plasma in the presence of argon gas, and the ionization threshold energy of atomic nitrogen (N) is less than that of argon at around 13 eV. So, the reason why molecular gases, such as nitrogen, require more power to produce a plasma is to make up for the many inelastic energy loss processes that occur for molecular gases such as vibrational and rotational excitation. §§§,***,††††

	100% Argon	99% Argon, 1% Nitrogen
Chamber base pressure (Torr)	1×10^{-7} Torr	1×10^{-7} Torr
Flow rate (sccm)	25.0 ± 0.25 sccm	24.7 ± 0.247 sccm, 0.3 ± 0.025 sccm
Anode voltage (V)	32 V	32 V
Anode current (A)	4.97 A	3 A
Keeper voltage (V)	32 V	36.3 V
Keeper current (A)	6 A	6 A
Heater voltage (V)	3.47V	16.34 V
Heater current (A)	0.59 A	2.49 A

Table 1: Cathode Operating Conditions for the Gas Mixing Experiment.

Comparable operating conditions were obtained for the case with and the case without nitrogen gas mixing. The anode voltage and keeper current were set values. The cathode was allowed to establish the anode current and keeper voltage difference needed to maintain the discharge due to the different gas flow compositions in the two cases.

Time-resolved Langmuir probe (LP) data was obtained on the centerline of the cathode's exit plane, 11.5-cm downstream (Fig. 2). The LP used for these experiments has a 0.127-mm probe tip diameter and a probe tip length of 1.27-mm (Fig. 3).

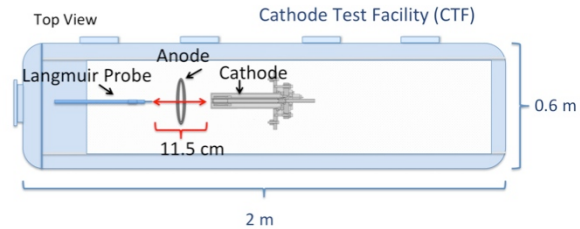


Fig. 2: Experimental Setup.

This figure shows how the cathode, anode and Langmuir probe (LP) were oriented in the Cathode Test Facility (CTF) for the gas-mixing tests.

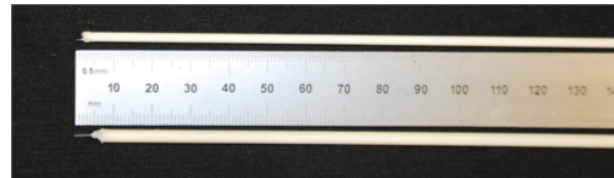


Fig. 3: Langmuir Probe.

The upper Langmuir Probe in this photograph, with a 0.127-mm diameter and 1.27-mm length, was used in this experiment.

IV. INITIAL GAS-MIXING RESULTS

Method of Analysis

Figure 4 shows the time-resolved Langmuir probe traces obtained for the point 11.5-cm downstream of the cathode's exit plane. The trace used for this analysis is shown as the red line in the first row of plots in this figure. It is the result of smoothing a combination of 25 traces taken from the first 25 negative-sloping sides ($dV/dt < 0$) of the triangle waveform used to obtain the LP traces. The combination of these traces without smoothing is shown as the black line in these plots. The bottom diagrams in this figure show the 25 Langmuir probe traces used for the two cases.

This way of combining the traces to still obtain time-resolved results was possible due to the fact that the 1-MHz sampling rate is not evenly divisible by the 15-kHz triangle wave used to obtain the Langmuir probe characteristics, and the analog nature of the function generator used to create the voltage sweeps. This caused the set voltage values from one sweep cycle to the next to be staggered. Combining 25 of these traces resulted in an effective voltage resolution of about 0.08 V for both cases. This resolution allowed more precise electron energy distribution functions (EEDFs) to be obtained in the analysis than what would have resulted

with just one of these traces. Some of the data points in the combined trace did have the same voltage value to the 4th digit after the decimal place. For these point pairs, which amounted to less than 4% of the total number of data points in both cases, the average of the current values was used in their place.

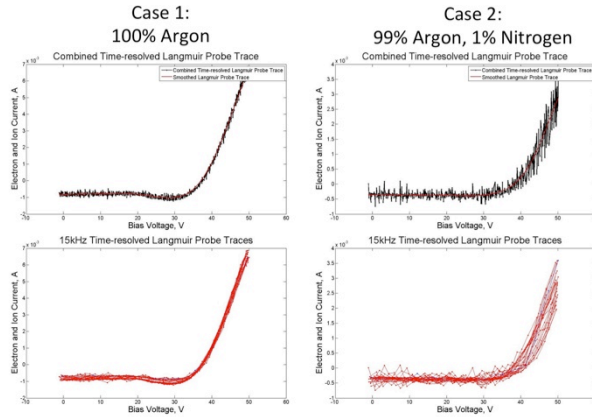


Fig. 4: Time-resolved Langmuir probe (LP) traces.

The plots on the second row of this figure show the first 25 Langmuir probe traces taken from the negative-sloping side ($dV/dt < 0$) of the sweep for the two gas-mixing cases. In the top row, the black line in these plots shows the time-resolved LP characteristic that results from combining these 25 traces. The red line is the smoothed version of this trace, which is what was used in the analysis.

	100% Argon	99% Argon, 1% Nitrogen
Floating Potential V_f (V)	37.0161 V	38.0959 V
Electron Temperature T_e (eV)	3.1928 eV	3.5526 eV
Plasma Potential V_s (V)	46.1882 V	48.2535 V
Number Density N_e (m^{-3})	$4.9597 \times 10^{17} m^{-3}$	$2.1912 \times 10^{17} m^{-3}$

Table 2: Gas Mixing Plasma Parameter Results.

Various calculated plasma parameters for the cases with and without nitrogen gas are shown in this table. The electron temperatures were calculated by taking the inverse slope of the natural log of the electron current portion of the I-V trace. This standard way of calculating T_e assumes a Maxwellian electron energy distribution.

Note: The full calculated values are shown. These values are still subject to the same error bars that are generally associated with plasma parameters calculated from Langmuir probe data.^{††††}

The LP traces form the plasma with nitrogen gas contained a considerable amount of additional noise, as

also seen in Figure 4. This noise may in part be due to the additional inelastic processes that are present for molecular gases, which begin at a low threshold energy of 0.02 eV and result in the vibrational and rotational excitation of the nitrogen molecules.^{†††} However, the process of combining multiple traces and smoothing the resulting trace produced an LP I-V characteristic that was easily analyzable.

Analysis of Results and Comparison to the Literature

The results showed that when 1% of the total gas flow is substituted with nitrogen, there is a slight increase in electron temperature from 3.2 eV to 3.6 eV along with an increase in plasma potential from 46 V to 48 V. In addition, the number density decreases from $5 \times 10^{17} m^{-3}$, with a pure argon plasma, to $2.2 \times 10^{17} m^{-3}$ (Table 2). This is consistent with experimental results obtained by Bai et al. They used an inductively coupled plasma (ICP) and took measurements 16-cm downstream with an rf-compensated LP. Bai et al. found that when the gas composition was 83.3% Ar and 16.7% N_2 , the electron number density, N_e , decreased from $4.0 \times 10^{16} m^{-3}$ to $2.0 \times 10^{16} m^{-3}$ and that the electron temperature remained relatively unchanged at 2.8 eV.^{§§§§} In tests carried out by, Pu et al. where argon made up 83.3% of an Ar- N_2 ICP plasma, they also saw a decrease in N_e from $2.7 \times 10^{17} m^{-3}$, with a pure argon plasma, to $1.7 \times 10^{17} m^{-3}$.^{†††} In addition, the electron temperature, T_e , increased from 3.4 eV to 3.8 eV.

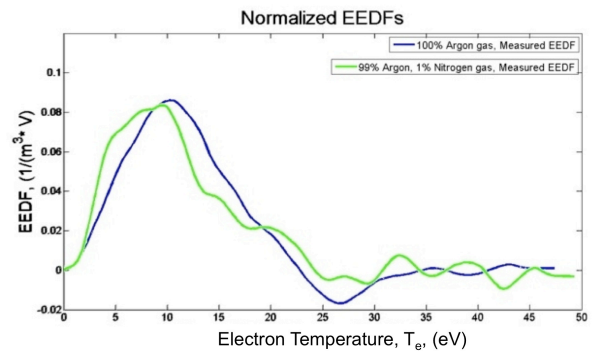


Fig. 5: Time-resolved EEDFs.

This diagram compares the EEDF for the case with a pure argon plasma, to that with 1% nitrogen. For the case with nitrogen gas, there are more distinct plateaus and peaks in the EEDF.

In the results from this experiment, for the case with 1% nitrogen mixing in an argon plasma, the EEDF contains more distinct plateaus and peaks. In Figure 5, we see that there are more low energy electrons and less intermediate energy electrons for this case as compared to the pure argon case. This may be due to the additional inelastic processes that lead to vibrational and rotational excitation of nitrogen molecules, which decreases the translational energy of various electron populations.

As also seen in Figure 5, in the high-energy region between 20 eV and 30 eV there are more electrons for the case with nitrogen mixing. Pu et al. point out that in a pure nitrogen plasma, a high energy tail can be generated by nitrogen in metastable states that are involved in superelastic collisions.^{†††} The presence of more electrons in the high energy tail of the EEDF for the case with N₂ mixing may be due to the influence of metastable states, which Gorse et al. demonstrate can have an effect on the overall mixture even when that gas is a very small percentage of the total flow.^{***}

In terms of the overall trends that were observed in the plasma parameters, there are still additional factors and additional results in the literature that need to be taken into account when formulating an explanation for their observation. In taking a closer look, both Pu and Bai took Langmuir probe measurements for several percentages of nitrogen mixing from 0% to 100% with a fixed total pressure and a fixed amount of power to the plasma source across all the cases tested. Pu et al. saw a trend of increasing T_e, from 3.4 eV to 4.2 eV, as the nitrogen concentration was increased from 0% to 100%. They reason that a lower N_e decreases the electron-electron collision frequency, which allows more electrons to reach higher velocities, thus increasing the average electron temperature. However, even though Bai et al. saw relatively no change in T_e when the nitrogen content was increased from 0% to 16.7%, as stated previously, they saw an overall trend of decreasing T_e from 2.8 eV to 2.2 eV as the nitrogen concentration was increased from 0% to 100%. Therefore, further investigation and comparison is needed to determine what other conditions and factors differed between these nitrogen-mixing experimental setups that affected electron temperature.

Due to the differing results in the trend for electron temperature with increasing nitrogen gas concentration, and because Pu et al.'s explanation for their observed increase in T_e relies on the decrease in N_e, this explanation can not be used to also explain the decreases in T_e seen in Bai et al.'s experiment since they also calculated a decrease in electron number density. In addition, how do we explain the decrease in number density that was seen in all of these experiments? If we assume quasi-neutrality^{****,††††}, then there was an equivalent decrease in the ion number density.

How can this decrease therefore be explained taking into account the properties of nitrogen relative to argon? As stated earlier, N₂ and Ar have comparable ionization threshold energies, the ionization cross-section of nitrogen is larger than that of argon by almost an order of magnitude, nitrogen has a relatively low dissociation energy under the conditions in this plasma mixture, and the ionization threshold energy of N is less than that of argon.^{****,††††,§§§§} (These cross-sections take into

account the fact that nitrogen atoms and molecules have a much smaller atomic mass than argon.) So, all of these properties would lead to an increase in ion number density and therefore an increase in N_e as well, which is not what was observed. Therefore, other processes must have a more dominant influence on the plasma properties, which lead to the significant decrease in N_e that was seen in all of these experiments. Noting that N_e decreased by about half when only 1% to 16.7% of the argon was substituted for nitrogen further supports this assertion. These electron loss processes most likely include superelastic collisions involving metastable states, and the additional modes that can result from inelastic processes in this mixture such as vibrational and rotational modes in the nitrogen molecules. These factors may also account for the different trends in T_e seen in the different experimental setups, because other aspects of each setup besides the pressure and total power may influence these processes.

Further analysis will make use of diagnostics to determine the number densities of the various populations of ions and neutrals and the ion energy distribution function in order to draw more detailed conclusions about the factors involved in the observed trends in electron number density and temperature. Unlocking these mysteries will bring us closer to understanding the complex nature of the interaction between the neutrals, ionized particles and electrons in this plasma which we can then build on to determine how to control them in these and even more complex systems such as the Hall thruster.

V. CONCLUSION & NEXT STEPS

Initial gas mixing tests that were carried out using a LaB₆ cathode showed that the plasma parameters and EEDF can be changed with the introduction of nitrogen gas into an argon plasma discharge. However, the additional factors that contribute to the observed changes and the reasons why they produce these observations need to be determined in order to be able to use this method to achieve predictive control of the plasma parameters and the EEDF.

The next steps for this research will include trying other percentages of nitrogen mixing, measuring the density and energy of the various gas and ion species to obtain more insight into the types of collisions that are taking place, and considering other combinations of gases. In addition, we will look at the frequency spectrum of the discharge current and voltage of the cathode for the various mixtures to see what oscillatory modes dominate and how they are changed by these different ratios of the two gases. Also, more spatial data points will be collected for each case and the plasma parameters derived from a couple of different sweep and sampling rates will be compared.

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